Math 347: Practice Exam 2

1. Let $(a_n)_{n\geq 1}$ be a sequence of real numbers.

   a) State the definition of a_n the limit of a_n being L.

   b) Suppose that $a_n = \frac{1}{n}$. What is $\lim a_n$? Prove your answer using the definition above.

   c) Suppose that $b_n = \frac{1}{n}\sum_{i=1}^n a_i$. Prove that b_n converges (Hint: The inequality from the previous midterm $\sum_{i=1}^n c_i| \leq \sum_{i=1}^n |c_i|$ might be useful).

2. Let $(a_n)_{n\geq1}$ be a sequence of real numbers.

   (a) State the definition of $(a_n)_{n\geq1}$ being a Cauchy sequence.

   (b) Provide an example of $(a_n)_{n\geq1}$ that is a Cauchy sequence.

   (c) Suppose that $(a_n)_{n\geq1}$ is a Cauchy sequence, prove that $(a_n)_{n\geq1}$ is bounded.

3. Consider sets S, T and R.

(a) State the definition of "S and T have the same cardinality".

(b) Suppose S and T have the same cardinality, and T and R have the same cardinality. Prove that S and R have the same cardinality (State precisely which property of composition of functions you are using. You don't have to prove this property though.).

(c) State the definition of $|S| = n$. Suppose that $|T| = m$. Let R be the set of functions between S and T. Prove that $|R| = |T|^{|S|}$.

4. Suppose that $x_1 = 1$ and $2x_{n+1} = x_n + \frac{3}{x_n}$.

(a) Prove that the limit of x_n exists.

(b) Find the limit $\lim_{n\to\infty} x_n$.